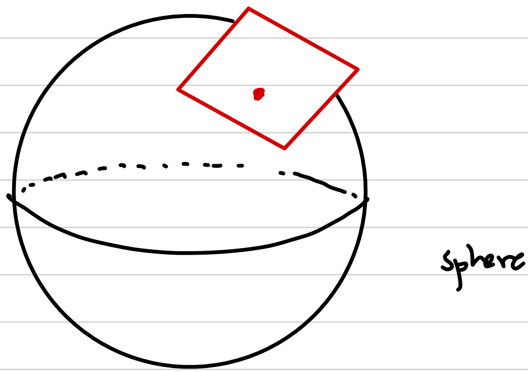
Regular Surfaces

<u>Ex.</u> To keep in mind....



- no sharp edges or points
- no self-intersections
- every pt. has a well-defined tangent plane.

" Reminder " : The derivative of a map $f: \mathbb{R}^n \to \mathbb{R}^m$

component functions

Sps. $f: \mathbb{R}^n \to \mathbb{R}^m$

Ex: $f(x,y) = (x\cos y,\ 2xy,\ e^x y^2)$

in $\mathbb{R}^n$

Defn We say f is differentiable at $\bar{p}$ if there is a linear map $df_p : \mathbb{R}^n \to \mathbb{R}^m$ (called the derivative of f) such that

$$\lim_{\bar{q} \to \bar{p}} \frac{\left| f(\bar{q}) - [f(\bar{p}) + df_p(\bar{q}-\bar{p})]\right|}{|\bar{q}-\bar{p}|} = 0 \quad \Big\} (*)$$

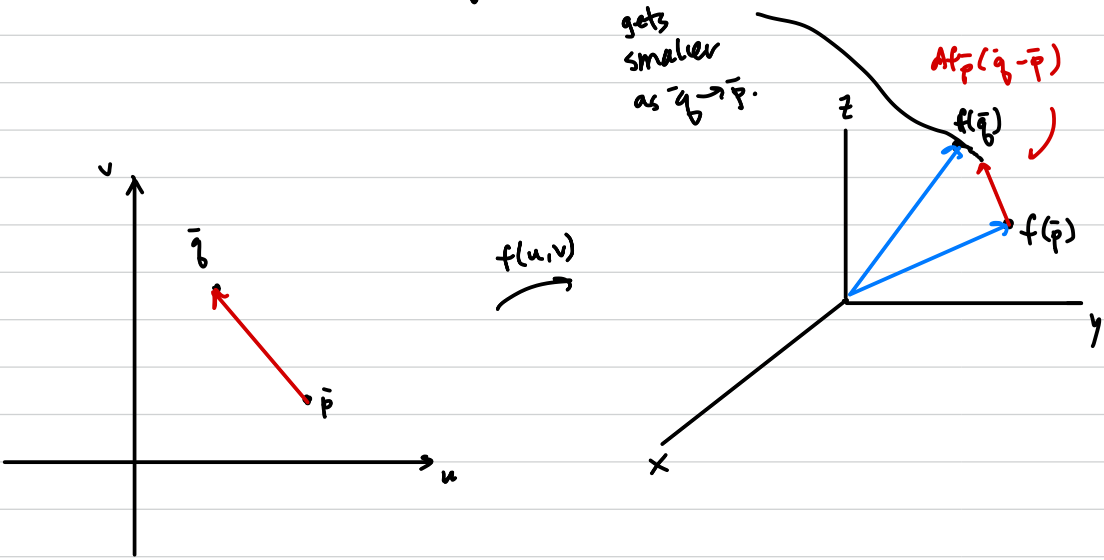
$(*)$ says: $f(\bar{p}) + df_p(\bar{q}-\bar{p})$ is a very good approximation of $f(\bar{q})$ as $\bar{q} \to \bar{p}$.

Key idea: $df_{\bar{p}}$ is a linear map that maps vectors based at $\bar{p}$ to vectors based at $f(\bar{p})$.

tangent vectors

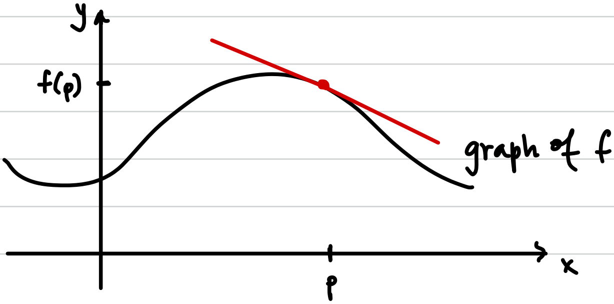
Visual:

$$\lim_{\bar{q} \to \bar{p}} \frac{\left| f(\bar{q}) - [f(\bar{p}) + df_{\bar{p}}(\bar{q} - \bar{p})] \right|}{|\bar{q} - \bar{p}|} = 0 \quad \Big\} (*)$$



$df_{\bar{p}}$ is a linear approximation of f near $\bar{p}$.

Similar (actually same): $f: \mathbb{R} \to \mathbb{R}$



tangent line is a good approx of graph of f near $(p, f(p))$.

Exercise: reconcile.