Thm (Fundamental Thm of Space Curves)

existence

1. Given smooth functions $\kappa(s) > 0$ and $\tau(s)$, $s \in I$, there exists a regular curve $\bar{\alpha} : I \to \mathbb{R}^3$ s.t. s is arc length, $\kappa(s)$ is curvature, and $\tau(s)$ is torsion of $\bar{\alpha}$.

uniqueness

2. Any other curve $\bar{\beta}$ satisfying the same conditions differs from $\bar{\alpha}$ by a rigid motion, ie. there is an orthogonal 3×3 matrix A with positive determinant and a 3-vector $\bar{c}$ s.t.

maintains orientation.

$$\bar{\beta}(s) = A\bar{\alpha}(s) + \bar{c}$$

Notes on rigid motions:

Rigid motions of $\mathbb{R}^3$ are maps $\mathbb{R}^3 \to \mathbb{R}^3$ that

preserve lengths and angles $\longleftarrow$ * geometry *

a.k.a. isometries of $\mathbb{R}^3$

orthogonal matrices are square matrices that

preserve the dot product:

$$A\bar{x} \cdot A\bar{y} = \bar{x} \cdot \bar{y} \quad \text{for all } \bar{x}, \bar{y}$$

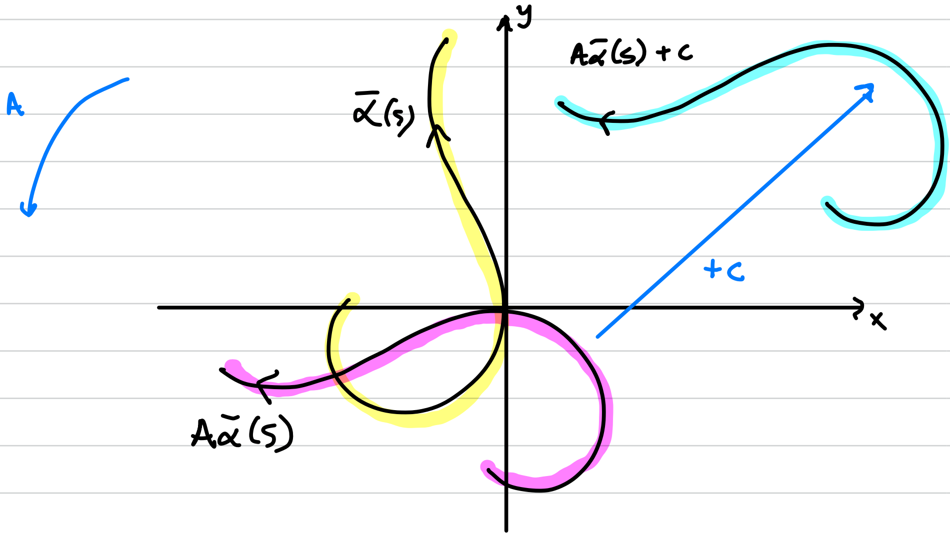
* preserve lengths and angles.

But any linear transformation fixes the origin. Adding

a constant vector $\bar{c}$ translates — also preserves lengths and angles.

(in $\mathbb{R}^2$, for simplicity) "maps to"

So: a picture of $\bar{\alpha}(s) \longmapsto A\bar{\alpha}(s) + \bar{c}$



So: thm says that if $\kappa_\beta(s) = \kappa_\alpha(s)$ and $\tau_\beta(s) = \tau_\alpha(s)$ for all s, then $\beta(s)$ is a rotation of $\bar{\alpha}(s)$ followed by a translation.

["unique up to rigid motion".

proof of thm: depends on major theorem of ordinary differential eqns.
↳ existence and uniqueness of soln of ODE. (we'll skip)