Let's consider $\bar{t}(s)$, $\bar{n}(s)$, and $\bar{b}(s)$ a bit more closely.

$\kappa(s) = |\bar{\alpha}''(s)|$

<u>Ex.</u> Curvature at any point on a straight line?

$|\bar{v}| = 1$

$$\bar{\alpha}(s) = \bar{p} + s\bar{v} = (p_1 + sv_1,\ p_2 + sv_2,\ p_3 + sv_3)$$

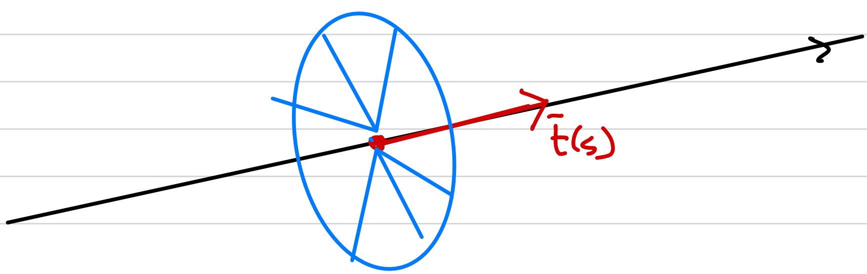
$\bar{t}(s)$

$$\bar{\alpha}'(s) = (v_1, v_2, v_3) \sim \bar{v}$$

$$\bar{\alpha}''(s) = (0, 0, 0)$$

$$\kappa(s) = |\bar{\alpha}''(s)| = 0.$$

Note: here, in $\mathbb{R}^3$, don't have an unambiguous choice for $\bar{n}(s)$.

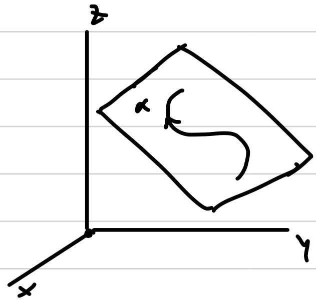


Prop: Sps $\bar{\alpha}$ is p.b.a.l., with $\kappa \neq 0$. Then $\bar{b}(s)$ is constant if and only if $\bar{\alpha}$ is planar.

(i.e. $\underset{\text{for all } s}{\bar{b}'(s) = 0} \Leftrightarrow \bar{\alpha}$ is planar.)

proof: ($\Leftarrow$) Sps. $\bar{\alpha}$ is planar.

(NTS: $\bar{b}'(s) = 0$ for all s.)



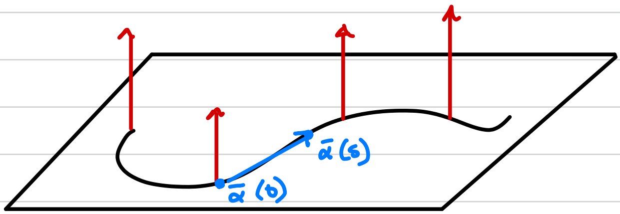
Since $\bar{\alpha}$ is planar, $\bar{t}(s)$ (i.e. $\bar{\alpha}'(s)$) lies in that plane for all s, and there is no change in $\bar{t}(s)$ $\perp$ to the plane.

Thus $\bar{\alpha}''(s)$ lies in plane as well, so $\bar{n}(s)$ lies in plane as well.

~~But $\bar{n}(s)$~~

Thus, $\bar{b}(s) = \bar{t}(s) \times \bar{n}(s)$ must be a unit vector $\perp$ to the plane containing $\bar{\alpha}$. So $\bar{b}(s)$ is constant, $\bar{b}'(s) = \bar{0}$.

($\Rightarrow$) Sps $\bar{b}(s)$ is constant, $\bar{b}(s) \equiv \bar{q}$ with $|\bar{q}| = 1$.



NTS: $\bar{\alpha}$ is planar.

Let $f(s) = (\bar{\alpha}(s) - \bar{\alpha}(0)) \cdot \bar{b}(s)$.

Then, by hw:

$$f'(s) = (\underbrace{\bar{\alpha}'(s)}_{t(s)}) \cdot \bar{b}(s) + (\bar{\alpha}(s) - \bar{\alpha}(0)) \cdot \underbrace{\bar{b}'(s)}_{\bar{0}}.$$

$$= \bar{t}(s) \cdot \bar{b}(s) + 0$$

$$= 0$$

$f(0) = (\bar{\alpha}(0) \underbrace{) - \bar{\alpha}(0)}_{\bar{0}}) \cdot \bar{b}(s)$

So: $f(s)$ is constant.

But $f(0) = 0$ So $f(s) = 0$ for $\underline{\underline{\text{all}}}$ s.

ie. $\bar{\alpha}(s) - \bar{\alpha}(0) \perp \bar{b}(s)$ for all $\bar{s}$. I.e. $\bar{\alpha}$ is planar. ✓