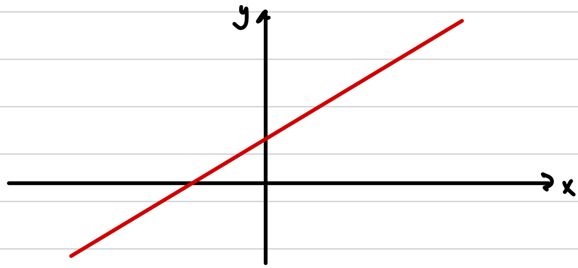
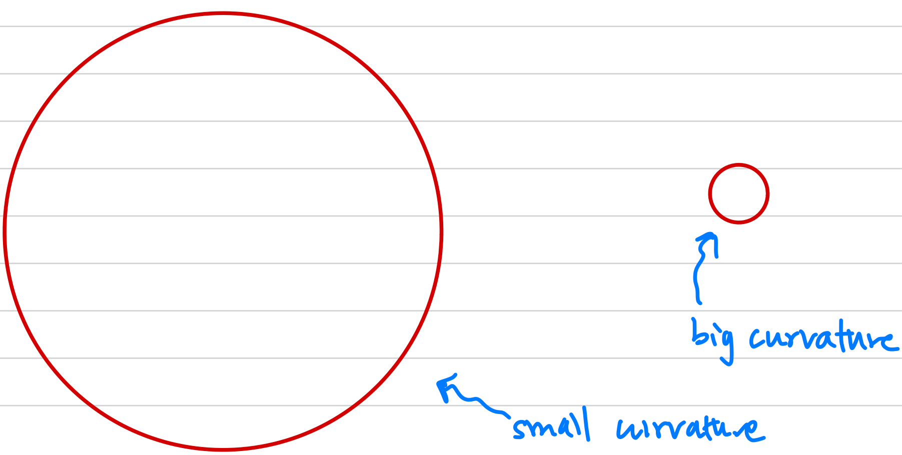
Curvature (for curves in $\mathbb{R}^2$ ... brief intuitive introduction)

Interpretation 1:

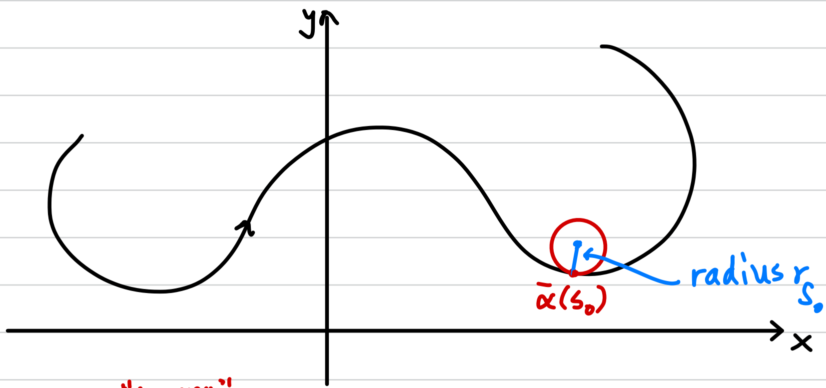
A straight line in $\mathbb{R}^2$ (or $\mathbb{R}^3$) has 0 curvature.



A circle, radius r, has curvature $\frac{1}{r}$.



For a general <u>plane</u> curve, $\bar{\alpha}: I \to \mathbb{R}^2$,



"kappa"

the curvature $\kappa(s_0)$ at $\bar{\alpha}(s_0)$ is given by

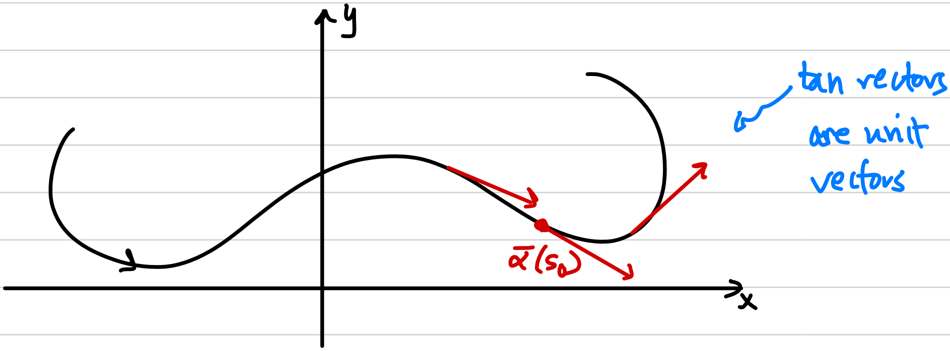
$$\frac{1}{r_{s_0}}$$

where r_{s_0} is the curvature of the "inscribed" circle.

a limiting process....
similar to finding tan line

Interpretation 2:

Sps. $\bar{\alpha}$ p.b.a.l.



Curvature at $\bar{\alpha}(s_0)$ measured by rate of change of tangent vector $\bar{\alpha}'(s)$ at $s = s_0$

$$\kappa(s) = |\bar{\alpha}''(s)|$$

unsigned curvature

(for plane curves, can assign a sign value)

Note: It can be shown that Interpretations 1 and 2 agree.