If $\bar{\alpha}$ is regular, we can always reparametrize by arc length.

theoretical. (computationally complicated)

How?

First: recall FTC:

variable

$$\frac{d}{dt}\int_a^t f(u)\,du = f(t)$$

In our situation, we know:

variable

arc length function

$$s(t) = \int_{t_0}^{t} |\bar{\alpha}'(u)|\,du$$

fixed base pt.

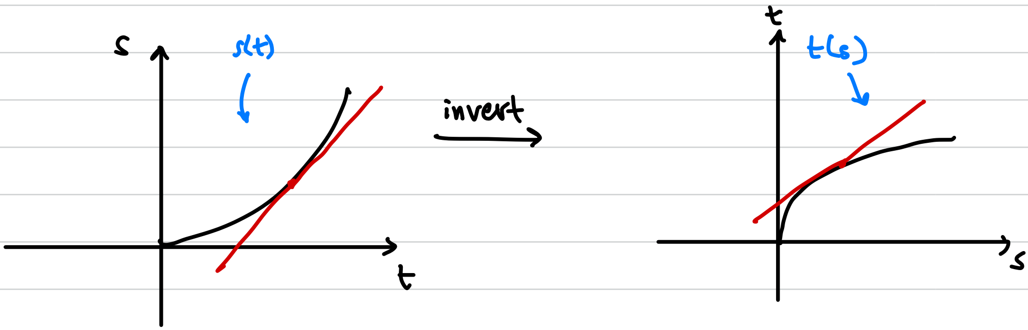
* b/c $\bar{\alpha}$ regular param

So, by FTC, $\dfrac{ds}{dt} = |\bar{\alpha}'(t)| \neq 0$ for all t

This means, since $s'(t) \neq 0$ for any t, that

$s(t)$ is monotonically increasing for all t

and thus is 1-1 and



Note that at corresponding pts

$$\frac{dt}{ds} = \frac{1}{\underbrace{\frac{ds}{dt}}_{|\bar{\alpha}'(t)|}}.$$

$\bar{\beta} = \bar{\alpha} \circ t$

Define $\bar{\beta}(s) = \bar{\alpha}(t(s))$. (NTS: $|\bar{\beta}'(s)| = 1$ for all s)

Then

$$|\bar{\beta}'(s)| = |\bar{\alpha}'(t(s))\, t'(s)| = |\bar{\alpha}'(t)|\left|\frac{dt}{ds}\right| = |\bar{\alpha}'(t)| \frac{1}{|\alpha'(t)|} = 1.$$

chain rule!

So, arc length of trace of $\bar{\beta}$ at time q is:

$$\int_0^q |\bar{\beta}'(s)|\, ds = \int_0^q 1\, ds = s\Big|_0^q = q - 0 = q.$$ yay!

* NOTE: When curve is param by arc length, tangent vector always has length 1. *