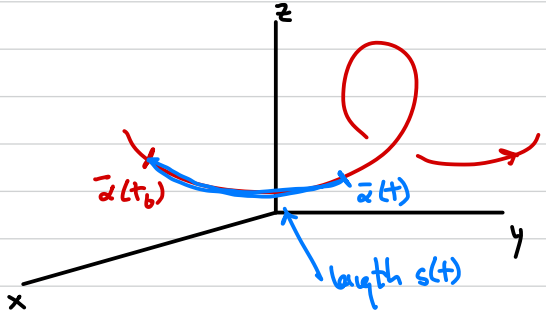
Arc Length

Defn Given a regular parametrized curve $\bar{\alpha}: I \to \mathbb{R}^3$

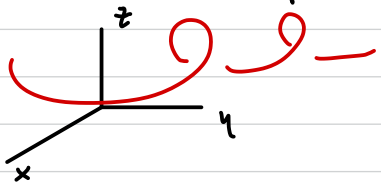
the **arc length** of $\bar{\alpha}$ starting from t_0 is

$$s(t) = \int_{t_0}^{t} |\bar{\alpha}'(u)| \, du.$$



Note: typically hard to compute arc length.

Recall: there are many ways to parametrize a given (trace of) curve in space ⇝



But, turns out: arc length is indep of parametrization (even though the defn seems to depend on the parametrization).

b/c of chain rule

(arc length function is well-defined)

We'll mainly consider curves that are parametrized by arc length.