





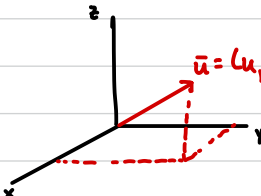
Can use the inner product (a.k.a. dot product) on  $\mathbb{R}^n$

to measure lengths and angles.

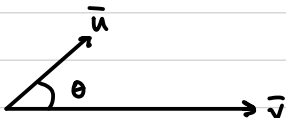
$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + \dots + u_n v_n$$

\*  \*  \*  \*  \* geometry

Length:


$$|\vec{u}| = \sqrt{u_1^2 + u_2^2 + u_3^2}$$
$$= \sqrt{\vec{u} \cdot \vec{u}}$$

Angles:



$$\vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta$$

$0 \leq \theta \leq \pi$

$$\cos^{-1} \left( \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} \right) = \theta$$

Angle b/w two curves intersecting at a point is angle b/w tangent vectors at that point.