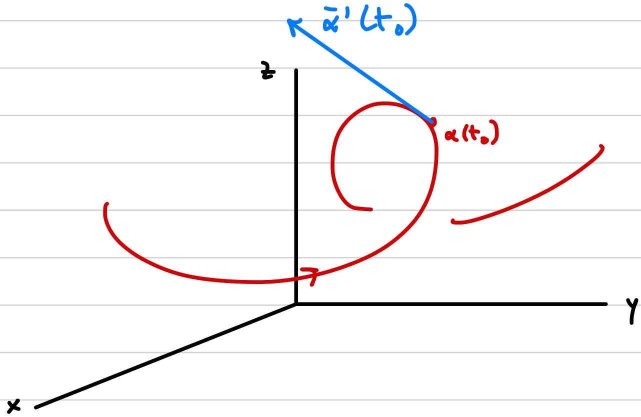
A <u>tangent vector</u> to a smooth curve $\bar{\alpha}(t)$ at t_0 is given by

$$\bar{\alpha}'(t_0) = (x'(t_0), y'(t_0), z'(t_0))$$



(consider $\bar{\alpha}(t) = (t^3, t^2)$ vs $\bar{\beta}(t) = (t, |t|)$, both at $t_0 = 0$.)

We say a smooth curve $\bar{\alpha}$ is <u>regular</u> if $\bar{\alpha}'(t) \neq \bar{0}$ for all t.

If $\bar{\alpha}'(t) = \bar{0}$, t_0 is called a <u>singular point</u>.

Ex. Line is regular. when parametrized as (pt) + t(dir vector)

i.e. (x_0+ta, y_0+tb, z_0+tc)

$\alpha'(t) = (a, b, c) \neq \bar{0}$

Note: Tangent vector depends on parametrization.

(exercise: Consider $\bar{\alpha}(t) = (t, t^2)$ at $t=1$

and $\bar{\beta}(t) = (2t, 4t^2)$ at $t = 1/2$.)

$(2t, (2t)^2)$

Finally, a curve $\bar{\alpha}(t)$, $t \in [a, b]$, is <u>closed</u> if

$\bar{\alpha}(a) = \bar{\alpha}(b)$

$\bar{\alpha}'(a) = \bar{\alpha}'(b)$

$\bar{\alpha}''(a) = \bar{\alpha}''(b)$

$\vdots$

... for all derivatives.