Curves

a.k.a. diff'ble (differentiable)

<u>Defn</u> A <u>parametrized, smooth curve</u> in $\mathbb{R}^3$ is a mapping (i.e. a function)

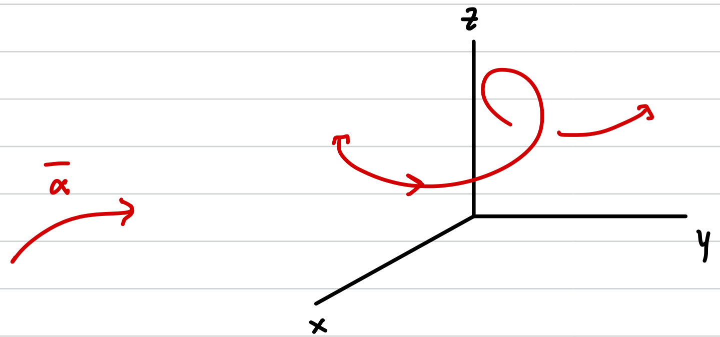
$$\bar{\alpha} : I \longrightarrow \mathbb{R}^3$$

interval

where curve lives... can be $\mathbb{R}^n$, any n.

$$\bar{\alpha}(t) = (x(t), y(t), z(t)), \text{ where } t \in I,$$

such that each of the component functions has derivatives of all orders.



$\bar{\alpha}$ is <u>smooth</u> if $x(t)$, $y(t)$, $z(t)$ have derivatives of all orders.

The variable t is called the <u>parameter</u>.

The image of $\bar{\alpha}$ in $\mathbb{R}^3$ (or $\mathbb{R}^n$) is called the <u>trace</u> of $\bar{\alpha}$.

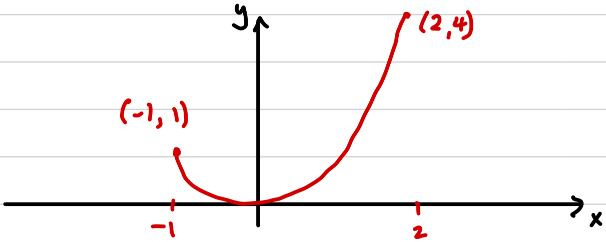
$x \quad y \quad$ here $y = x^2$

$\bar{\alpha}(t) = (t, f(t))$

$f(t) = t^2.$

Ex. $\bar{\alpha}(t) = (t, t^2)$, $t \in [-1, 2]$

picture?

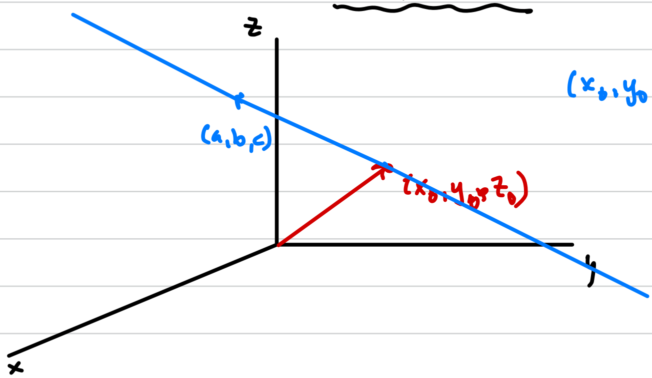


smooth?

yes!

Ex Lines: determined by a point (x_0, y_0, z_0)

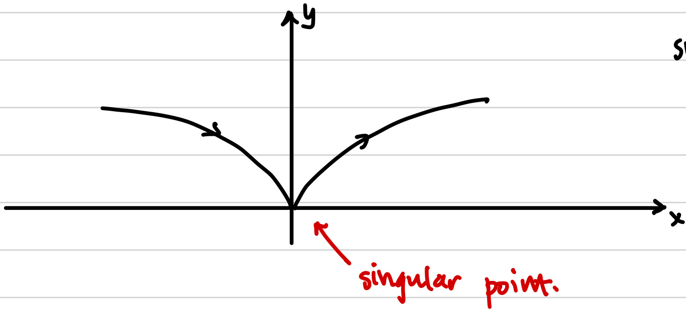
and a direction vector (a, b, c).



$(x_0, y_0, z_0) + t(a, b, c)$, $t \in \mathbb{R}$

$\bar{\alpha}(t) = (x_0 + ta,\ y_0 + tb,\ z_0 + tc).$

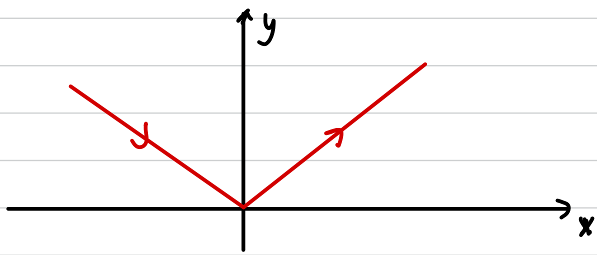
Ex $\vec{\alpha}(t) = (t^3, t^2)$, $t \in \mathbb{R}$



smooth? yes!

Ex $\vec{\beta}(t) = (t, |t|)$, $t \in \mathbb{R}$

picture?



smooth? No! $\rightsquigarrow$ $|t|$ not diff'ble at $t=0$.