Math 335: Differential Geometry
Homework 9 (due November 27)

1. Compute the Christoffel symbols for an open set of the plane
   (a) In cartesian coordinates.
   (b) In polar coordinates.

   Use Gauss' equation to compute K in both cases.

2. Suppose that $(\bar{x}, U)$ is a coordinate chart such that $F = 0$. Confirm that:

$$K = -\frac{1}{2\sqrt{EG}}\left(\frac{\partial}{\partial v}\left(\frac{E_v}{\sqrt{EG}}\right) + \frac{\partial}{\partial u}\left(\frac{G_u}{\sqrt{EG}}\right)\right).$$

3. Verify that the surfaces

$$\bar{x}(u,v) = (u\cos v, u\sin v, \ln u)$$
$$\bar{y}(u,v) = (u\cos v, u\sin v, v)$$

   have equal Gaussian curvature at the points $\bar{x}(u,v)$ and $\bar{y}(u,v)$ but that the mapping $\bar{x} \circ \bar{y}^{-1}$ is not an isometry. This show that the "converse" of the Gauss' Theorema Egregium is not true.

4. Explain why no neighborhood of a point in a sphere may be isometrically mapped into a plane. This shows that there is no perfect way to create a map of any part of the world. No matter how you make your map, there will always be distortion.

5. Show that there exists no surface which has a chart $\bar{x}(u,v)$ that satisfies $E = G = 1$, $F = 0$, and $e = 1$, $g = -1$, $f = 0$.