

Math 335: Differential Geometry

Homework 8 (due November 13)

1. Define Gaussian curvature for a nonorientable surface. Can you define mean curvature for a nonorientable surface?
2. Show that at the origin  $(0, 0, 0)$  of the hyperboloid  $z = axy$  we have  $K = -a^2$ .
3. Let  $H$  denote the mean curvature:  $H(p) = \frac{1}{2}(\kappa_1 + \kappa_2) = -\frac{1}{2}\text{trace}(dN_p)$ . Let  $K$  denote the Gaussian curvature. Prove that  $H^2 \geq K$ .
4. An asymptotic curve on a surface  $S$  is a curve such that the normal curvature in the direction of the tangent to the curve is 0 at any point on the curve. Determine the asymptotic curves of the catenoid

$$\bar{x}(u, v) = (\cosh v \cos u, \cosh v \sin u, v).$$

5. Show that a regular curve  $\bar{\alpha}(s)$  parametrized by arc length is an asymptotic curve if and only if  $\bar{N}'(s)$  is orthogonal to  $\bar{\alpha}'(s)$  for all  $s$ .
6. Show that if the mean curvature is zero at a nonplanar point, then this point has two orthogonal asymptotic directions.