Math 335: Differential Geometry
Homework 5 (due October 16)

1. Let $S \subset \mathbb{R}^3$ be a regular surface, and suppose that $\bar{p}_0$ is a point in $\mathbb{R}^3$ not contained in S. Let $d : S \to \mathbb{R}$ be given by $d(\bar{p}) = |\bar{p} - \bar{p}_0|$; that is d is the distance from $\bar{p}$ to a fixed point $\bar{p}_0$ not in S. Prove that d is a differentiable function on S.

2. Suppose that $f : \mathbb{R}^3 \to \mathbb{R}$ is a differentiable function. Suppose that $S \subset \mathbb{R}^3$ is a regular surface. Show that $f_{|S}$ (f restricted to S) is a differentiable function from S to $\mathbb{R}$.

3. Let $S_1 \subset \mathbb{R}^3$ be a regular surface and let S_2 denote the xy-plane in $\mathbb{R}^3$. Here, we are considering S_2 to be a regular surface in $\mathbb{R}^3$, given by the set $\{(x, y, z) \in \mathbb{R}^3 \,|\, z = 0\}$. Suppose that $\pi : S_1 \to S_2$ is the map which takes each $p \in S$ to its orthogonal projection in S_2. In other words, if $p = (x, y, z)$, then $\pi(p) = (x, y, 0)$. Is π a differentiable map between surfaces?

4. Prove that the definition of a differentiable map between surfaces does not depend on the parametrizations chosen.

5. Describe the region of the unit sphere covered by the image of the Gauss map of the following surfaces:

   (a) Paraboloid $z = x^2 + y^2$.

   (b) Hyperboloid $x^2 + y^2 - z^2 = 1$ (nuclear power plant).

   (For this problem, you can start by drawing pictures to figure out the regions, but ultimately, find parametrizations and compute $\bar{N}(\bar{p})$ for points on the surface and argue why your pictures are correct.)

6. Use the definition of a differentiable map between surfaces to show that for a regular surface S, the Gauss map $\bar{N} : S \to S^2$ is differentiable.